An Ordered Array of Terminated Metallic Posts as an Embedding Network for Lumped Microwave Devices

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Abstract—An ordered array of metallic posts is proposed as an embedding network for lumped devices at microwave frequencies. The individual metallic posts, terminated in a lumped device, extend across a parallel-plane transmission line excited by the TEM mode. In the analysis the impedance discontinuity for an individual terminated metallic post in a transmission line bounded on two sides by electric walls and two sides by magnetic walls is evaluated. Design concepts useful in employing these results with a two-dimensional array of metallic posts terminated with semiconductor devices are described.

I. Introduction

During the past fifteen years numerous discrete semiconductor devices have been developed to perform a variety of functions at microwave frequencies [1]. In this period most of the device and related circuit work has involved optimization of single semiconductor device components. However, fundamental power handling and practical bandwidth constraints exist with single device components [2]; these limitations become particularly restrictive at higher microwave frequencies as the size of the discrete device must be reduced. These constraints have served as an impetus to the investigation of circuit embodiments that can be utilized to combine numerous discrete semiconductor devices.

In this paper a novel embedding network, consisting of an ordered array of metallic posts terminated with lumped devices, is described. Initially the advantages and disadvantages of various network configurations are described (Section I). The problem of evaluating the transmission properties of the generalized ordered array proposed is then reduced to the determination of the impedance discontinuity of a single terminated post under various imposed conditions (Section II). The impedance discontinuity of this single terminated post is then obtained using the suitable Green's function (Section III). Finally design concepts useful in employing the resultant discontinuity impedance in the design of two-dimensional arrays of metallic posts terminated with semiconductor devices are described (Section IV).

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Hines [3] has classified these network techniques into the federal and unified. The *federal* technique uses many single device circuits interconnected by corporate-feed networks. The optimum single device component can then be utilized, with an arbitrarily large number of components being combined. This approach is conceptually straightforward, limited in application by electrical phase errors and mechanical complexity in the corporate feeds [4]. The *unified* technique embeds many devices in a large network which is not separable into individual complete circuits. Thus an optimized single device component cannot be directly implemented. Only a minimal effort has been expended investigating unified networks, principally because of the complex interaction between device operation and circuit design.

In many instances the unified technique can be subdivided into two classifications: longitudinal combining and transverse combining. In *longitudinal* combining the devices are placed along the direction of energy propagation. One example is the use of many p-i-n diodes in a filter structure to obtain a broad operating bandwidth [5]. Another is the periodic placement of many loosely coupled amplifiers along a transmission line to obtain high gain [3]. In *transverse* combining numerous devices are placed in a plane perpendicular to the direction of energy propagation. An example is the location of many p-i-n diodes in a circular configuration between inner and outer conductor of a coaxial line to achieve a higher power switch [6].

With both unified techniques a minimal number of devices has been utilized (typically 3 to 5 with active devices or 10 to 30 with control devices). With longitudinal combining the structures investigated can be limited by the complexity of mounting the individual devices, variations in individual device characteristics, and loss in the adjacent microwave circuitry. With transverse combining the number of devices that can be mounted is limited by the physical size of the transmission line; the number of devices that can be embedded is reduced as the operating frequency is increased and the guide cross section is reduced to prevent multimode propagation. Therefore, the unified configurations investigated to date are limited both conceptually and practically in the number of discrete devices that can be utilized.

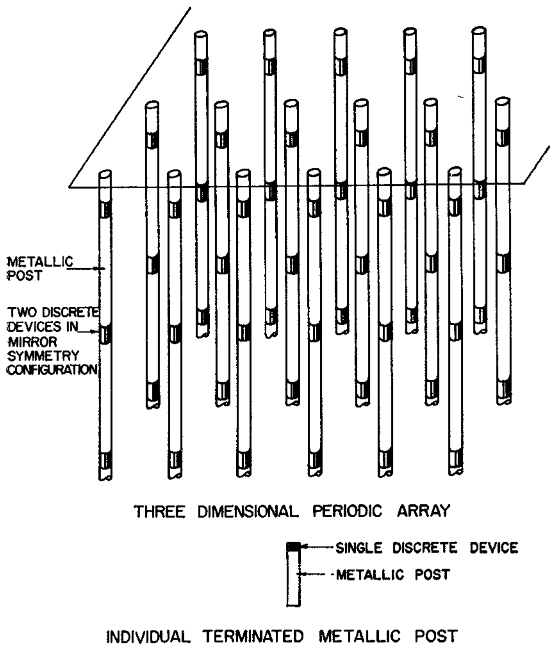

Fig. 1. Three-dimensional terminated metallic post ordered array.

As a result of these considerations, and others based upon semiconductor device characteristics, some desirable features of unified circuits have evolved.

1) To incorporate a large number of devices, the circuit mounting should be capable of batch processing.

2) To ease constraints on semiconductor materials and processing procedures, the circuit should not require identical characteristics from all discrete devices for successful operation.

3) To obtain high-power capability, many devices must be mounted in a transverse combining configuration.

4) To maximize the average power capability with a minimal number of discrete devices, the devices should be located in contact with a good heat sink.

5) To allow amplification or switching functions, dc biasing should be possible. To allow frequency conversion or parametric amplification, a different frequency ac signal should be capable of being applied to the discrete devices.

6) To examine the effect of various circuit impedances on device operation, a method of circuit tuning is desirable.

A particular configuration does not have to satisfy all these desirable features in order to be useful for a given component. However, the listing provides a basis for selecting circuit configurations worthy of investigation.

In a general context a unified network consists of an ensemble of discrete semiconductor devices, with metallic and/or dielectric inclusions, in either a mixture or ordered array. An arbitrarily large number of discrete devices can be utilized by simply fabricating an appropriate volume of the unified network to be placed in a suitable transmission medium. Within this general description of unified networks, the choice of circuit configurations is extensive. The three-dimensional periodic array of terminated metallic posts depicted in Fig. 1 was chosen as satisfying the general goals outlined.

II. Method of Analysis

A. Equivalent Transmission Line

The transmission properties of the three-dimensional ordered array can be obtained by analyzing an equivalent periodically loaded transmission line, bounded on two sides by electric walls and two sides by magnetic walls as shown in Fig. 2. If electric walls can be placed perpendicular to the terminated metallic posts and magnetic walls parallel to these obstacles without affecting the field configuration in the three-dimensional array, the impedance discontinuity of a row of obstacles can be obtained by analyzing an individual obstacle in this transmission line. The impedance calculated by this approach is identical to the discontinuity introduced by a row in the original three-dimensional array if the electric and magnetic walls inserted do not disturb the original field configuration.

This technique of inserting electric and magnetic walls to isolate a one-dimensional periodic array has been invoked in analyzing transmission properties of artificial dielectrics without discussion of the generality of the method [7]–[9]. However, it can be shown that these walls can be inserted if the following sufficient conditions are satisfied.

1) The material supporting the obstacles can be described by a relative permittivity and relative permeability which are isotropic and homogeneous.

2) These planes could be inserted without disturbing the field configuration that would exist in the support material with the obstacles removed.

3) The current density induced on the obstacles has the following symmetry. With respect to the magnetic walls, the x component of current density is an even function, the y component is odd, and the z component even; with respect to the electric walls, the x component of current density is an even function, the y component odd, and the z component odd.

If a plane wave linearly polarized in the x direction and propagating in the z direction is incident upon the terminated metallic posts supported by a homogeneous, isotropic material, these conditions will be satisfied. As a result an analysis of the one-dimensional periodic array will yield transmission characteristics identical to that obtained with the original three-dimensional periodic array. In the one-dimensional periodic array, an electric wall can be placed parallel to, and midway between, the two electric walls without disturbing the field configuration. With this additional wall inserted, a periodic array

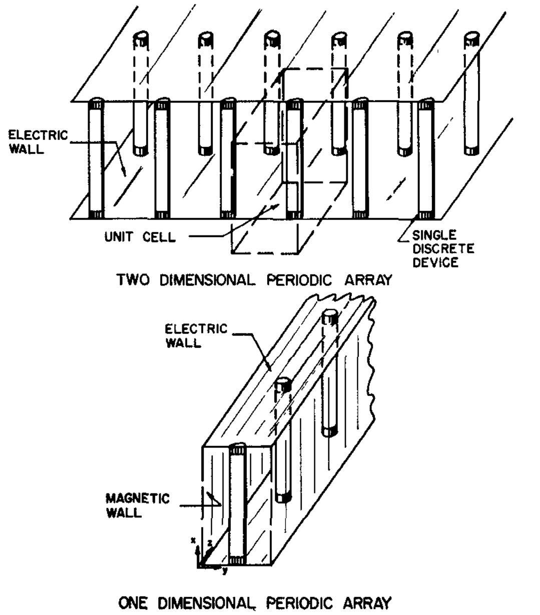

Fig. 2. Two-dimensional and one-dimensional terminated metallic post ordered arrays.

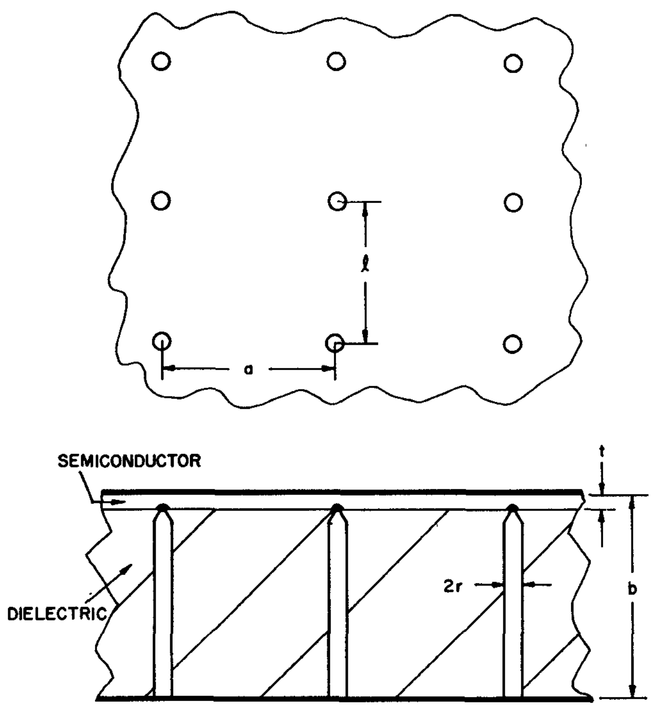

Fig. 3. Practical two-dimensional ordered array.

of singly terminated metallic posts has been isolated.

A practical configuration actually implemented is shown in Fig. 3. The discrete devices are located between the end of the metallic posts and the electric wall (ground plane). The remainder of the semiconductor can be retained as illustrated for ease of mounting. To prevent multimode propagation the dimensions of the equivalent transmission line must be constrained. For the single terminated metallic post the guide height b must be less than one-half wavelength while the guide width a must be less than one wavelength. With these constraints imposed, the analysis of the multidimensioned ordered array of terminated metallic posts has been reduced to an analysis of a one-dimensional periodic array.

It should be noted that the experimental work has emphasized control components in which shallow junctions are diffused into high-resistivity material. Therefore the configuration illustrated in Fig. 3 can be easily implemented over the entire microwave spectrum without having enhanced attenuation in the transmission line. With active arrays of devices fabricated from n–n^+ epitaxial material, this enhanced attenuation must be considered since the effective wall conductivity may be low.

In obtaining the transmission characteristics of the ordered array, emphasis is placed in this manuscript on obtaining the equivalent circuit of an individual terminated metallic post in the one-dimensional array (or equivalently the discontinuity of a row of terminated metallic posts in the two-dimensional array). The development of the transmission characteristics of periodic loaded lines after the periodic discontinuity impedance has been determined is straightforward and well documented [13]. Interaction between adjacent discontinuities can be considered as a perturbation on these results, as will be described in Section IV.

B. Appropriate Green's Function

A two-dimensional Green's function was used in determining the equivalent circuit for the terminated post. This method of analysis has been employed by Lewin [10] in determining the equivalent circuit of a probe in rectangular guide. Lewin's theoretical results have been used successfully in evaluating parameters of a single discrete semiconductor device mounted on a post in rectangular guide [11], [12].

After the Green's function is obtained, many of the mathematical manipulations required are similar to those used by Lewin. In this manuscript differences between the analysis for an incident TEM mode in the transmission line depicted in Fig. 2 and the analysis for an incident TE_{10} mode in rectangular guide will be emphasized. For convenience in comparing these analyses, the notation utilized by Lewin is used throughout (CGS practical system of units).

The electric and magnetic fields that are supported by the transmission line with the terminated post configuration can be derived from the electric-type Hertzian vector π satisfying the equations

$$\nabla^2\pi + k^2\epsilon\pi = \frac{120\pi j}{k\epsilon} I(x)\mathbf{1}_x\delta(z)\delta(y-d) \qquad (1)$$

where

$$E = \nabla(\nabla\cdot\pi) + k^2\epsilon\pi, \qquad \text{with } \mu = 1.0$$

$$H = (jk\epsilon/300)\nabla\times\pi, \qquad \text{with } k = \frac{2\pi}{\lambda}\cdot \tag{2}$$

It is assumed that the post is of sufficiently small diameter that a filamentary current located at the center of the post will adequately satisfy the boundary conditions. Thus the net field at the surface of the post is independent of radial position, although an axial variation is possible. Dissipation in the dielectric used to support the array is also neglected, a condition usually achieved to a reasonable degree.

It can be shown by using (2) with Maxwell's equations that

$$\pi = 1_x\pi_x = 1_x \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{mn}\cos\left(\frac{n\pi y}{a}\right)\cdot\cos\left(\frac{m\pi x}{b}\right)e^{-\Gamma_{mn}|z|} \tag{3}$$

where

$$\Gamma_{mn} = \left(\frac{m^2\pi^2}{b^2} + \frac{n^2\pi^2}{a^2} - k^2\epsilon\right)^{1/2}.$$

satisfies the boundary conditions at the walls of the guide.

The two-dimensional Green's function $G(x, y, z\,|\,x'=\zeta, y'=d, z'=0)$ can be obtained in the usual manner:

$$\begin{aligned} G(x, y, z\,|\,x' &= \zeta, y' = d, z' = 0)\\ &= 1_x \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} -\frac{j240\pi\epsilon_{mn}}{abk\epsilon\Gamma_{mn}}\cos\left(\frac{n\pi d}{a}\right)\cos\left(\frac{n\pi y}{a}\right)e^{-\Gamma_{mn}|z|}\left(k^2\epsilon - \frac{m^2\pi^2}{b^2}\right)\cos\left(\frac{m\pi x}{b}\right)\cos\left(\frac{m\pi\zeta}{b}\right)\\ &+ 1_y \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} -\frac{j240\pi\epsilon_{mn}}{abk\epsilon\Gamma_{mn}}\left(\frac{n\pi}{a}\right)\cos\left(\frac{n\pi d}{a}\right)\sin\left(\frac{n\pi y}{a}\right)e^{-\Gamma_{mn}|z|}\left(\frac{m\pi}{b}\right)\sin\left(\frac{m\pi x}{b}\right)\cos\left(\frac{m\pi\zeta}{b}\right)\\ &+ 1_z \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} -\frac{j240\pi\epsilon_{mn}}{abk\epsilon\Gamma_{mn}}\cos\left(\frac{n\pi d}{a}\right)\cos\left(\frac{n\pi y}{a}\right)\left[\frac{\partial}{\partial z}e^{-\Gamma_{mn}|z|}\right]\left(\frac{-m\pi}{b}\right)\sin\left(\frac{m\pi x}{b}\right)\cos\left(\frac{m\pi\zeta}{b}\right) \end{aligned} \tag{4}$$

where

$$\epsilon_{mn} \equiv \begin{cases} 1, & m \neq 0, \quad n \neq 0\\ \frac{1}{2}, & m = 0 \text{ and } n \neq 0 \text{ or } n = 0 \text{ and } m \neq 0\\ \frac{1}{4}, & m = n = 0. \end{cases}$$

III. Obtaining an Equivalent Circuit

With this appropriate Green's function, an equation can be written that satisfies the tangential boundary condition at the surface of the post with the fundamental mode incident, an equation similar to that utilized by Lewin [10]:

$$\begin{aligned} 1 + \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} -\frac{j240\pi\epsilon_{mn}}{abk\epsilon\Gamma_{mn}}\cos\left(\frac{n\pi d}{a}\right)\cos\left(\frac{n\pi}{a}(d \pm r)\right)\\ \cdot\left(k^2\epsilon - \frac{m^2\pi^2}{b^2}\right)\cos\left(\frac{m\pi x}{b}\right)\int_0^b I(\zeta)\cos\left(\frac{m\pi\zeta}{b}\right)d\zeta\\ = I(x)Z_1\delta(x) + I(x)\frac{\sqrt{30/\sigma\lambda}}{r} \end{aligned} \tag{5}$$

where the incident electric field of unity amplitude is taken to be linearly polarized in the x direction, the post conductivity is equal to σ, and the impedance Z_1 is taken to be spatially lumped at $x=0$, resulting in a voltage drop $Z_1I(0)$, where $I(0)$ is the current flow at the end of the post. It should be noted that circuit losses are evaluated since a finite post conductivity is included. Experimentally such losses will be determined principally from surface considerations and contact resistances. However, dissipation in the post can limit the uppermost frequency at which the terminated metallic post is practical, so that its inclusion is desirable. The effect on the equivalent shunt reactance from the metallic inductive surface is negligible except near resonance, so that the assumption of a purely resistive surface impedance is reasonable and will not limit the usefulness of these results. It should be recalled that a lossless support dielectric and infinite wall conductivity have been assumed, so that wall and dielectric losses must be added to the post dissipation evaluated in the analysis to obtain the total attenuation. The field evaluation at the surface of the post has been taken at the point $x=0$, $y=d+r$, $z=0$. Other points satisfying the equation $x^2+z^2+(y-d)^2=r^2$ could have been taken, where differences in the final results depend upon variation of the local fields over the post, a variation assumed negligible.

By multiplying (5) by $I(x)$, integrating over x, and employing the usual relation for the reflection coefficient with a shunt impedance Z, the following expression can be obtained:

$$Z = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}{}' \frac{j2Z_0\epsilon_{mn}}{k\epsilon^{1/2}\Gamma_{mn}} \cos\left(\frac{n\pi d}{a}\right)\cos\left(\frac{n\pi(d \pm r)}{a}\right) \cdot\left(k^2\epsilon - \frac{m^2\pi^2}{b^2}\right)\left[\frac{\int_0^b I(\zeta)\cos(m\pi\zeta/b)\,d\zeta}{\int_0^b I(\zeta)\,d\zeta}\right]^2 + \frac{I^2(0)Z_1}{(J/b)^2} + \frac{\sqrt{30/\sigma\lambda}}{r}\,\frac{\int_0^b I^2(x)\,dx}{(J/b)^2} \quad (6)$$

where

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}{}' F_{mn}(x) \equiv \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} F_{mn}(x) - F_{00}(x)$$

$$J \equiv \int_0^b I(x)\,dx$$

$$Z_0 = (120\pi/\epsilon^{1/2})(b/a). \quad (7)$$

For convenience a reactance parameter, X_m is defined as

$$-\sum_{n=0}^{\infty}{}'\left(\frac{240\pi}{abk\epsilon}\right)\frac{(m^2\pi^2/b^2 - k^2\epsilon)}{\Gamma_{mn}}\epsilon_{mn}\cos\left(\frac{n\pi d}{a}\right) \cdot\cos\left(\frac{n\pi(d \pm r)}{a}\right) \equiv \frac{X_m}{b^2} \quad (8)$$

where

$$\sum_{n=0}^{\infty}{}' F_{mn}(x) = \begin{cases} \sum_{n=1}^{\infty} F_{0n}(x), & m = 0 \\ \sum_{n=0}^{\infty} F_{mn}(x), & m \geq 1. \end{cases}$$

The reactance parameter for $m=0$, i.e. X_0, can be obtained from (7) and (8) as

$$X_0 = \frac{Z_0 a}{(\lambda/\epsilon^{1/2})}\left[\ln\left[\frac{a/2\pi r}{\sin(\pi d/a)}\right] + \sum_{m=1}^{\infty} 2\cos^2\left(\frac{m\pi d}{a}\right)\left[\frac{1}{(m^2 - k^2\epsilon a^2/\pi^2)^{1/2}} - \frac{1}{m}\right]\right]. \quad (9)$$

For the case of interest, $d=a/2$, (9) is identical to an equation derived by MacFarlane [14] for a post entirely across the guide. This result is expected, as with $m=0$ there are no modes excited with field variations in the x direction. This uniformity implies a discontinuity that is independent of the x direction, namely, the post entirely spanning the guide ($Z_1=0$). The terms represented by X_m ($m \geq 1$), which will evaluate to be capacitive, represent couling of the post into higher order modes with field variations in the x direction.

Returning to (8) which defines the general reactance parameter, the following series summation is indicated ($m \geq 1$):

$$\sum_{n=0}^{\infty}\epsilon_{mn}\frac{\cos(n\pi d/a)\cos(n\pi(d \pm r)/a)}{(m^2\pi^2/b^2 + n^2\pi^2/a^2 - k^2\epsilon)^{1/2}}. \quad (10)$$

A similar summation is equated in Lewin [15] utilizing a variant of Poisson's formula [16]. Using this approach and the small post constraint ($r \ll 2d$), the following can be obtained:

$$\sum_{n=0}^{\infty}\epsilon_{mn}\frac{\cos(n\pi d/a)\cos(n\pi(d \pm r)/a)}{\Gamma_{mn}} = \frac{a}{2\pi}\Big[K_0(r\Gamma_m) + K_0(2d\Gamma_m) + \sum_{n=1}^{\infty}\big[K_0((2na + r)\Gamma_m) + K_0((2na - r)\Gamma_m) + K_0((na + d)2\Gamma_m) + K_0((na - d)2\Gamma_m)\big]\Big] \quad (11)$$

where

$$\Gamma_m \equiv \left(\frac{m^2\pi^2}{b^2} - k^2\epsilon\right)^{1/2} \quad (12)$$

and K_0 is the modified Bessel function of zero order.

Separating the $m=0$ series from the double infinite series in (6), and using (8), a simplified expression for the shunt impedance is obtained:

$$Z = \frac{I^2(0)Z_1}{(J/b)^2} + \frac{\sqrt{30/\sigma\lambda}}{r}\,\frac{\int_0^b I^2(x)\,dx}{(J/b)^2} + jX_0 + \sum_{m=1}^{\infty} jX_m\left[\frac{\int_0^b I(\zeta)\cos(m\pi\zeta/b)\,d\zeta}{\int_0^b I(\zeta)\,d\zeta}\right]^2. \quad (13)$$

This expression is variational with respect to the current variation $I(x)$, so that an accurate expression for Z can be obtained if a reasonable estimate of $I(x)$ is made. However, a form for $I(x)$ has been tacitly assumed when spatially lumping Z_1 at the end of the post. Thus return to (5), separate the $m=n=0$ term from the double infinite series, use the definitions of the reactance parameter (8) and the characteristic impedance of the line (7), expand $I(x)$ in a Fourier cosine series, and evaluate the Fourier coefficients by multiplying by cos $(n\pi x/b)$ and integrating over x to obtain

$$I(x) = +\frac{b}{BD}\left[1 + 4Z_1\sum_{n=1}^{\infty}\frac{1 - \cos(n\pi x/b)}{jX_n + 2\sqrt{30/\sigma\lambda}\;b/r}\right] \quad (14)$$

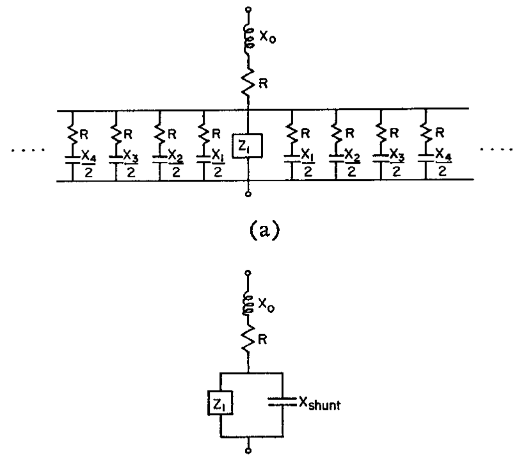

Fig. 4. Equivalent circuit of single terminated metallic post. (a) With any post conductivity. (b) With low-loss metallic post.

where

$$B \equiv -\frac{(Z_0/2 + jX_0 + \sqrt{30/\sigma\lambda}\ b/r}{b^2}$$

$$D \equiv \frac{Z_1}{B} - b^2 - 4Z_1 b^2 \sum_{n=1}^{\infty} \frac{1}{jX_n + 2\sqrt{30/\sigma\lambda}\ b/r}. \quad (15)$$

Note that the relative magnitudes of the Fourier coefficients for $n > 0$ are independent of Z_1, although the relative magnitude of the $n = 0$ coefficient (compared with the others) is a strong function of Z_1. As a result of the initial formulation expressed by (5), the spatial distribution of $I(x)$ along the post is independent of Z_1, except for the relative amplitude of the spatially uniform term.

Using (14) in the expression for the shunt impedance (13), the following is obtained:

$$Z = jX_0 + \sqrt{\frac{30}{\sigma\lambda}}\frac{b}{r} + \frac{1}{1/Z_1 + \sum_{n=1}^{\infty} [2/(\sqrt{30/\sigma\lambda}\ b/r + jX_n/2)]} \quad (16)$$

where X_0 is given by (9) and X_n by (8). This shunt discontinuity can be represented by the equivalent circuit shown in Fig. 4(a).

Having arrived at an equivalent circuit involving an infinite series involving X_m with $m \geq 1$, a tractable expression for X_m is obtained by using (11) in (8) and invoking the small post constraint ($r \ll 2d$) and symmetry condition ($d = a/2$):

$$X_m \approx -\frac{Z_0 \Gamma_m^2}{k\epsilon^{1/2}} \frac{a}{\pi} [K_0(r\Gamma_m) + 2K_0(a\Gamma_m)]. \quad (17)$$

Expression (17) shows that $|X_m/2|$ has a lower bound of

$$\frac{Z_0 \Gamma_m^2}{2k\epsilon^{1/2}} \frac{a}{\pi} K_0(r\Gamma_m).$$

If $b/(t+r) > m > 1$, $X_m/2$ will be greater than Z_0, while $X_1/2$ will be the same order of magnitude as Z_0 unless the guide height is close to one-half wavelength in the dielectric.[1] Since $\sqrt{30/\sigma\lambda}\ b/r$ is the resistance of a post completely spanning the guide, the numerical value must be much less than the guide impedance to prevent excessive circuit losses, that is, $Z_0 \gg \sqrt{30/\sigma\lambda}\ b/r$. Thus in general $X_m/2 \gg \sqrt{30/\sigma\lambda}\ b/r$ for all m in the range of interest so that to a high degree of approximation

$$Z = jX_0 + \sqrt{\frac{30}{\sigma\lambda}}\frac{b}{r} + \frac{1}{1/Z_1 + \sum_{m=1}^{\infty} (4/jX_m)} \quad (18)$$

with X_m given by (17). It should be noted that dissipation caused by the post resistance is still included in the equivalent circuit. The high Q of the higher order mode capacitive reactances are set to infinity, but the lower Q of the inductive reactance is maintained finite.

When the derived expression for X_m is used in the indicated series summation $\sum_1^\infty (4/jX_m)$, the series diverges. This divergence results from the erroneous evaluation of energy storage in the modes of sufficiently high order that an appreciable field variation exists across the discrete device at the end of the post. In fact for the idealized configuration employed, that is a finite diameter post terminated in an infinitesimal gap, the discrete device impedance would indeed be shorted by the infinite gap capacitance.

Examining the physical configuration in Fig. 3, it is apparent that the mathematical formulation of a constant diameter post terminated in an infinitesimally thin device is an inadequate representation for modes having appreciable spatial variations over the distance that the post diameter is not constant, namely, a distance of $t+r$. Thus the energy stored in these evanescent modes cannot be correctly evaluated with the model. The spatial variation is significant when $m \sim [b/(t+r)]$, where the brackets indicate "nearest integer to," so that the evaluation of X_m becomes meaningless for higher values of m. Therefore, it is deemed appropriate that the infinite series indicated in (18) be replaced by a truncated summation $\sum_1^{[b/(t+r)]} (4/jX_m)$. The upper limit depends upon the ratio of the guide height to the distance over which the post radius is changing and assumes that the post is tapered to make contact with the discrete devices as shown in Fig. 3.

[1] The justification for considering only a finite range of m will be apparent shortly.

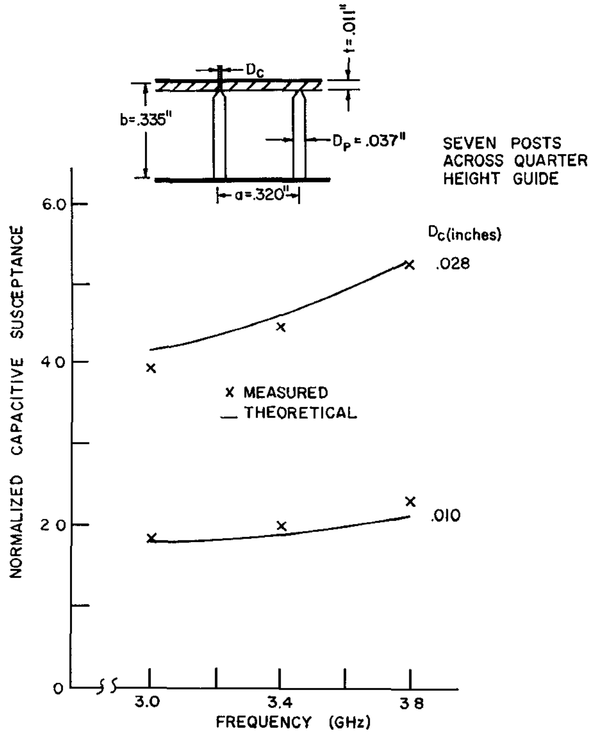

Fig. 5. Correlation of experimental results with theoretical calculations.

With an untapered post the upper limit would be $[b/t]$. The finite sum indicated is used to define a net shunt reactance X_{shunt}:

$$X_{\text{shunt}} \equiv \frac{1}{\sum_{1}^{[b/(t+r)]} (4/jX_m)}. \tag{19}$$

This analysis can be extended by considering a surface current instead of a filamentary current [17] and/or considering the lumped device to be spatially extended from 0 to t instead of lumped at $x=0$. The evaluation of X_{shunt} and the divergence considerations will differ. However, (19) has proven useful in the limited design examples attempted, as illustrated in Fig. 5. These measurements were taken in quarter-height S-band guide with four step quarter-wave transformers providing an impedance match to standard test equipment [18].

It is difficult to ascertain the accuracy of the general expression for X_{shunt}. In the cases analytically investigated and experimentally evaluated where X_{shunt} varied from the semiconductor device capacitive reactance to 5 times this capacitive reactance, agreement to ± 20 percent was obtained. However, this accuracy may be degraded if appreciable energy storage exists in modes with $m > [b/(t+r)]$.

In conclusion a shunt impedance equivalent circuit has been developed for a single terminated metallic post (depicted in Fig. 4(b)) in the transmission line bounded by two electric walls and two magnetic walls. For convenience, the equivalent circuit parameters are summarized as follows:

$$Z = jX_0 + \sqrt{\frac{30}{\sigma\lambda}}\,\frac{b}{r} + \frac{1}{1/Z_1 + \sum_{m=1}^{[b/(t+r)]} (4/jX_m)}$$

$$X_0 = \frac{Z_0 a}{\lambda/\epsilon^{1/2}}\left[\ln\left(\frac{a}{2\pi r}\right) + 2\sum_{m=1}^{\infty}\cos^2\left(\frac{m\pi}{2}\right)\cdot\left[\frac{1}{(m^2 - k^2\epsilon a^2/\pi^2)^{1/2}} - \frac{1}{m}\right]\right]$$

$$X_m = -\frac{Z_0\Gamma_m{}^2}{k\epsilon^{1/2}}\,\frac{a}{\pi}\left[K_0(r\Gamma_m) + 2K_0(a\Gamma_m)\right], \qquad m \neq 0$$

$$Z_0 = (120\pi/\epsilon^{1/2})(b/a).$$

In many applications a tuning element or another semiconductor device may be desirable at the other end of the post. The analysis is readily extended by adding a third term to the right-hand side of (5), namely, $I(x)Z_2\delta(x-b)$. The analysis proceeds in a similar fashion [18].

IV. Practical Design Factors

In utilizing this discontinuity impedance in the design of two-dimensional embedding networks with lumped semiconductor devices, two factors must be considered. First, the impedance requirements of an individual device must be considered in selecting the parameters of the transmission line bounded by electric and magnetic walls. Second, the effect of the row spacing must be considered on the overall impedance and transmission characteristics. In this section, these considerations are discussed.

Although detailed design concepts will depend upon the type of lumped device considered, some general factors can be stated for semiconductor diodes. The transmission line dimensions a and b should be kept small to minimize the series inductive reactance and maximize the shunt capacitive reactance, respectively. However, as the guide height b is reduced, dissipative loss in the electric walls increases and broad-band excitation becomes more difficult. As the guide width is decreased, the post radius is necessarily reduced, thereby reducing the Q of the post. A detailed design procedure useful for semiconductor control devices, based upon these principles, has been developed and verified experimentally [18]. The design for a parallel-plate transmission line is easily implemented in rectangular waveguide, in a manner utilized in the design of waffle-iron filters [18], [19].

It should be noted that any interaction between adjacent rows of these discontinuities has been neglected, although the electrical spacing may be a fraction of a wavelength. If the cross-sectional dimensions a and b of the transmission line are comparable to or less than the longitudinal spacing l, the interaction will be small. If interaction of adjacent discontinuities is significant, methods well described for artificial dielectrics can be invoked [13]. In impedance matching this periodically loaded medium to other circuitry, techniques described in the literature can be applied [13], [20].

This circuit embodiment should be useful between X and Q bands for many semiconductor devices. In this frequency range numerous devices can be fabricated on a semiconductor wafer while maintaining a reasonable electric spacing between the posts. Therefore, batch processing should be possible. Above Q band, metallic losses may be prohibitive, at least with control components. At frequencies below X band the structure could be utilized, but batch assembly may not be feasible due to the relatively large physical distance between devices.

V. Summary

The transmission characteristics of an ordered array of terminated metallic posts useful as an embedding network for lumped devices can be obtained by using the equivalent circuit developed and straightforward network techniques. Although the evaluation of the equivalent capacitive reactance shunting the discrete device impedance cannot be precisely determined with the analysis employed, the equivalent circuit is adequate for first-order design purposes. In cases where this capacitive reactance must be known accurately, experimental measurements must be used. The terminated post configuration should be useful with a variety of semiconductor components for microwave applications in the frequency range between X and Q band.

Appendix

List of Symbols

a	Periodic spacing of terminated metallic posts in the transverse direction.
b	Height of transmission line.
d	Position of terminated metallic post relative to magnetic wall ($d=a/2$).
f	Frequency.
j	$(-1)^{1/2}$.
k	Free-space propagation constant.
l	Periodic spacing of terminated metallic posts in the longitudinal direction.
r	Radius of metallic post.
t	Thickness of discrete device.
$G(x, y, z \mid x', y', z')$	Green's function with rectangular coordinates.
$I(x)$	One-dimensional current element in metallic post.
J	Post current density parameter $\equiv \int_0^b I(x)\,dx$.
$K_0(x)$	Modified Bessel function of zero order.
X_m	Reactance parameter defined by (8).
X_0	Reactance of post completely spanning guide.
X_{shunt}	Equivalent reactance shunting terminating impedance.
Z	Impedance of shunt discontinuity.
Z_0	Characteristic impedance of transmission line.
Z_1	Terminating impedance, between end of post and guide wall.
$\mathbf{1}_x$	Unit vector in x direction.
Γ_{mn}	Propagation constant of mn mode.
$\delta(x-c)$	Dirac delta function: $\int_{-\infty}^{\infty} f(x) \cdot \delta(x-c) = f(c)$.
ϵ	Material relative permittivity (assumed real).
ϵ_0	Dielectric permittivity of free space.
ϵ_{mn}	Function defined for integral m and n; equal to 1 if $m \neq 0$ and $n \neq 0$, equal to 1/2 if $m=0$ and $n \neq 0$ or $m \neq 0$ and $n=0$, equal to $\frac{1}{4}$ if $m=n=0$.
λ	Free-space wavelength.
μ	Material relative permeability (assumed equal to unity).
$\boldsymbol{\pi}$	Electric type Hertzian potential.
σ	Conductivity.

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Synthesis of Mixed Lumped and Distributed Impedance-Transforming Filters

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***Abstract*—The design of a class of impedance-transforming filters in the form of very compact and convenient mixed lumped and distributed ladder networks is presented. The synthesis utilizes the distributed prototype technique introduced in a previous paper, but here a new approximation function appropriate to the impedance transformer problem is derived. In addition to combining the properties of an impedance transformer and a low-pass filter, the new circuit represents a solution to the problem of short-line matching to an extreme impedance value without using extreme impedance values in the transformer. Broad-band designs are tabulated for a wide range of parameters. A discussion of the application of the technique in the design of mixed lumped and distributed broad-band matching networks is included.**

A 50–10-Ω transformer was designed for the band 3.5–7.0 GHz, having a voltage standing-wave ratio of 1.15 and giving an attenuation >20 dB in the band 10.5–21.0 GHz. The length of this transformer is 0.875 in, and the experimental results showed excellent agreement with theory.

Introduction

THE design of impedance transformers in a compact and convenient format has been the subject of considerable research effort. The conventional (and usually best) solution to the problem of matching between two purely resistive impedances is the well-known multiquarter-wave section stepped impedance transformer. However, this can be rather lengthy, especially at the lower microwave frequencies. The short-step impedance transformers described by Matthaei *et al.* [1] are, as the name implies, much shorter than the corresponding quarter-wave transformers. Their chief disadvantage is the fact that the range of impedance levels within any given transformer is considerably larger than the input and output impedance levels. For example, a six-section $\lambda_m/16$ transformer for a 5:1 impedance change and fractional bandwidth $\omega=0.8$ requires normalized impedances varying between 0.572 and 8.74, a range of 15.25:1, compared with only 5:1 for the transformer ratio obtained. Thus a transformer from 10 Ω to 50 Ω utilizing this design would require one line having the very low impedance value of 5.72 Ω.

This example illustrates the problem of producing short-length impedance transformers where the impedance levels within the transformer lie in a range encompassed by the terminating impedances. Preferably it would be most desirable to design a short transformer to an extreme terminating impedance without requiring such an impedance in the transformer. Conventional quarter-wave transformers achieve this at the expense of length.

In one sense a solution to the problem can be obtained with mixed lumped and distributed circuits. A simple way to see this is to note that we might consider replacing the $\lambda_m/16$ line of impedance 5.72 Ω in the Matthaei transformer described by a lumped capacitor. This can be accomplished rather accurately since the line is mainly capacitive. If all the low-impedance lines

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